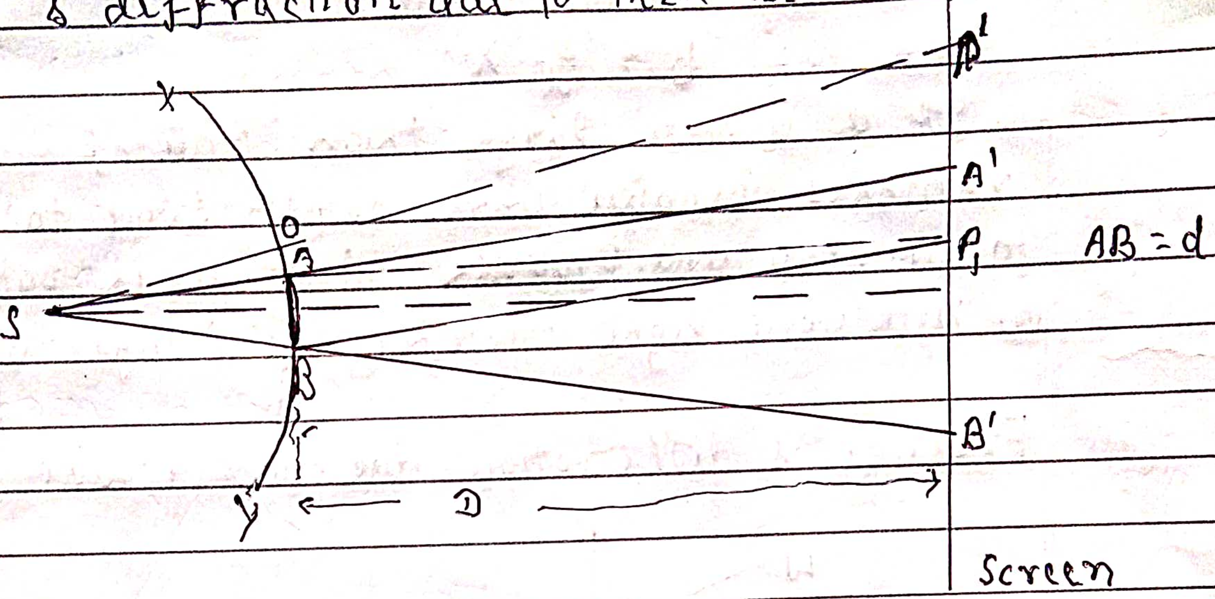
* Fresnel's diffraction due to thin wire & circular aperture:-

Fresnel's diffraction due to thin wire:-



Let AB represent a long narrow wire, perpendicular to plane of paper and placed exactly parallel to a long narrow slit S, which is illuminated by a light source of wavelength λ. AB represents diameter d of the wire. XY represents cylindrical wavefront & A'B' is geometrical shadow on the screen.

Let us consider a point P' outside the geometrical shadow. The effect of the wavefront at P' will be partly due to upper part of the wavefront i.e. above the pole O and partly due to the part OA contains an even no. of half period zones, the point P' will be dark, if it is odd P' will be bright. The effect at P' due to lower part negligible. Thus above A' and below B', there will be by diffraction bands. If wire is extremely thin (AB = d) then effect of part BY will also present and diffraction bands on either side of geometrical shadow will not be ~~dist~~ distinct.

Now we consider a point P_1 inside the geometrical shadow. Wavelets from A and B may reach P, in phase or out of phase. Hence equally spaced interference fringes are formed in geometrical shadow.

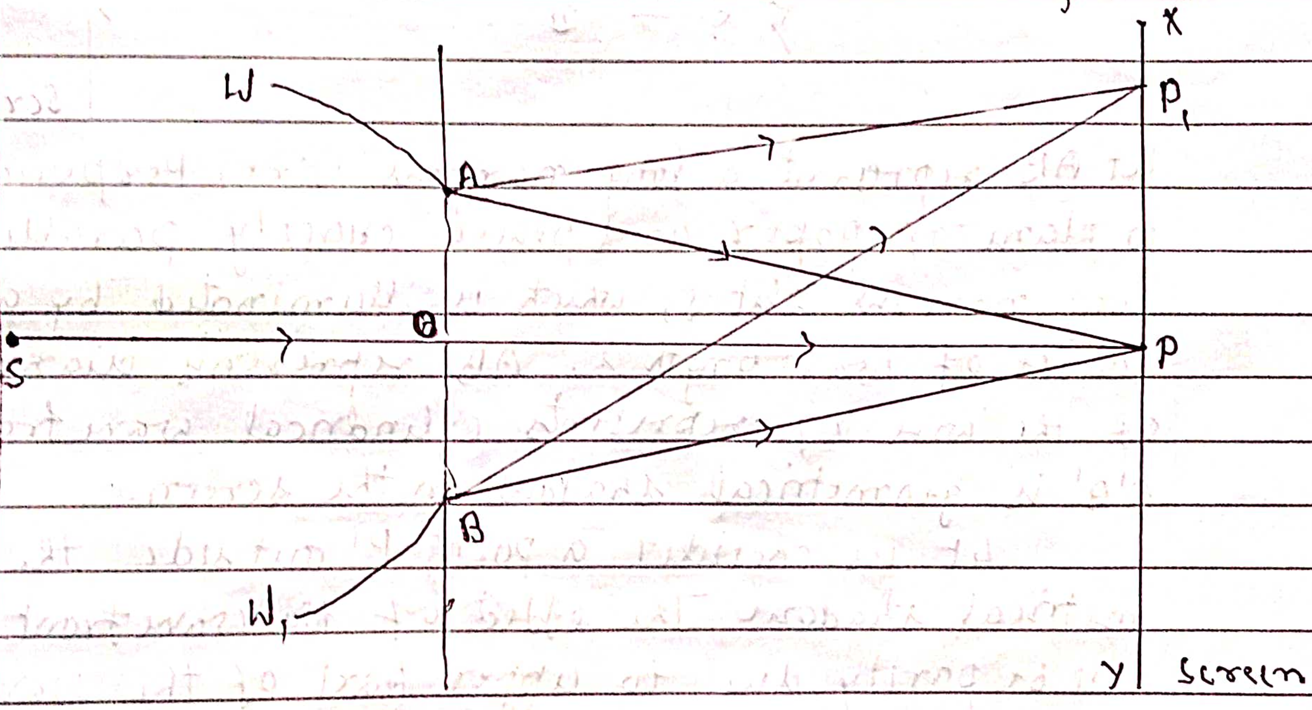
If B = Fringe width,

then $B = \frac{D}{d} \cdot \lambda$

(a) If d is very large, then interference fringes vanishes, because waves can't bend to such extent as to overlap and produce interference pattern.

(b) With white light interference fringes are coloured.

Fresnel's diffraction due to circular aperture:—



$SO = a$, $OP = b$, $OA = OB = r$, $PP_1 = x$

Let AB be a small circular aperture of centre O and radius r. S is a point source of wavelength λ. XY is a screen perpendicular to the plane of paper and P is a point such that SP is perpendicular to XY. WOW_1 is the incident spherical wave front. O is the pole of wave front with respect to P.

Intensity at P:— Fresnel's half period zones are constructed with P as centre and radii $b+\frac{\lambda}{2}$, $b+\frac{2\lambda}{2}$ etc. on the exposed wave front AOB. Depending upon the distance OP = b, the no. of half period zones may be even or odd. If aperture contains even no. of zones then they mutually cancel each other in pair and intensity at P will be less. If the no. is odd then intensity will be greater. If screen is moved then distance b changes and aperture will alternately contain odd and even no. of zones and hence point P will be alternately bright and dark.

Now path difference between marginal and central ray

$$\delta = (SA + AP) - SOP$$

$$= (a^2+r^2)^{1/2} + (b^2+r^2)^{1/2} - (a+b)$$

$$= a\left[1+\frac{r^2}{a^2}\right]^{1/2} + b\left[1+\frac{r^2}{b^2}\right]^{1/2} - (a+b)$$

$$= a\left[1+\frac{r^2}{2a^2}\right] + b\left[1+\frac{r^2}{2b^2}\right] - (a+b)$$

$$\therefore \delta = \frac{r^2}{2}\left[\frac{1}{a}+\frac{1}{b}\right]$$

$$\therefore r^2 = \frac{2ab\delta}{a+b}$$

Let the path difference $\delta = n.\frac{\lambda}{2}$, where n may be odd or even.

$$\therefore r^2 = \frac{abn\lambda}{a+b} \quad \text{or,} \quad b = \frac{ar^2}{nab - r^2}$$

Thus a+b changes, n alternately changes from odd to even, making successive intensity at P max^m & min^m.

Intensity at a point P_1 away from centre:—

let be path difference between the secondary waves from A and B and reaching P_1 can be given by

$$\delta = BP_1 - AP_1$$

$$= \{b^2+(x+r)^2\}^{1/2} - \{b^2+(x-r)^2\}^{1/2}$$

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$$\therefore \delta = b\left[1+\frac{(x+r)^2}{2b^2}\right] - b\left[1+\frac{(x-r)^2}{2b^2}\right]$$

$$= \frac{1}{2b}\left[(x+r)^2-(x-r)^2\right]$$

$$= \frac{1}{2b}\left[(x+r)^2-(x-r)^2\right]$$

$$= \frac{1}{2b}\cdot 4xr = \frac{2rx}{b}$$

P_1 will be dark, if $\delta = 2n\cdot\frac{\lambda}{2}$

$$\therefore 2n\cdot\frac{\lambda}{2} = 2r\cdot\frac{x_n}{b} \quad [x = x_n \text{ for nth order}]$$

$$\therefore x_n = \frac{nb\lambda}{2r}$$

Similarly if $\delta = (2n+1)\frac{\lambda}{2}$, P_1 will have bright fringe

$$\therefore (2n+1)\frac{\lambda}{2} = \frac{2rx_n}{b}$$

$$\therefore x_n = \frac{(2n+1)b\lambda}{4r}$$